Review and Report on Results of Leptonic Decays of D^+ and D_s^+ Mesons

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Abstract

In the last 25 years, many e^+e^- experiments and fixed-target experiments performed to search for and study the leptonic decays of the D^+ and D_s^+ mesons. By 2012, more than 530 signal events of the D^+ leptonic decays and about 4×10^3 signal events of the D_s^+ leptonic decays have been accumulated at these experiments. With these leptonic decay signal events, both decay constants f_{D^+} and $f_{D_s^+}$ are, respectively, measured at an accuracy level of 2.4% and 1.6%, which can be used to more precisely test the LQCD calculations of the decay constants. Comparing these precisely measured f_{D^+} and $f_{D_s^+}$ with those predicted with theories based on QCD provides some information about New Physics beyond the Standard Model. In addition to these, with the measured branching fractions for $D^+ \to l^+\nu$ and $D_s^+ \to l^+\nu$ decays, the CKM matrix elements $|V_{\rm cd}|$ and $|V_{\rm cs}|$ can be determined. Comparing these $|V_{\rm cd}|$ and $|V_{\rm cs}|$ to those determined from the CKMfitter or extracted from D meson semileptonic decays can also provide some information about the New Physics. In this article, we review and report the results on the leptonic decays of D^+ and D_s^+ mesons measured at different experiments. For the results which have already been published, we review these in shorter summaries, while for the results which have not been published or have not been reported yet before we report these with more detailed discussion.

1 Introduction

In the SM (Standard Model) of particle physics, the $D^+_{(s)}$ (through this article, charge conjugation is implied) meson can decay into $l^+\nu_l$ (where l is e, μ or τ) through a virtual W^+ boson as shown in Fig. 1. The decay rate is determined by the wavefunction overlap of the two quarks at the origin, and is parameterized by the $D^+_{(s)}$ decay constant, $f_{D^+_{(s)}}$. All strong interaction effects between the two quarks in initial state are absorbed into $f_{D^+_{(s)}}$. To the lowest order, as the analogue of the decay width of $\pi^+ \to l^+\nu_l$, the decay width of $D^+_{(s)} \to l^+\nu_l$ is given by [1]

$$\Gamma(D^+_{(s)} \to l^+\nu_l) = \frac{G_F^2 f^2_{D^+_{(s)}}}{8\pi} \mid V_{\rm cd(s)} \mid^2 m_l^2 m_{D^+_{(s)}} \left(1 - \frac{m_l^2}{m^2_{D^+_{(s)}}}\right)^2, \quad (1)$$

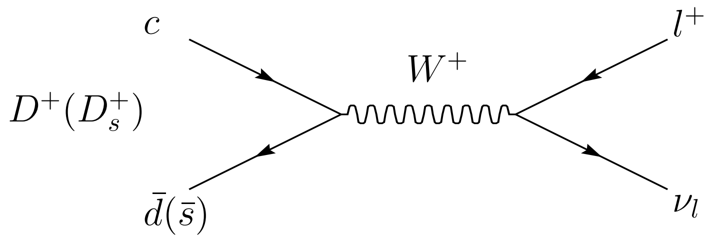

Figure 1: The decay diagram for $D_{(s)}^+ \to l^+\nu_l$.

where G_F is the Fermi coupling constant, $V_{\rm cd(s)}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between the two quarks $c\bar{d}(\bar{s})$ [2] in $D^+(D_s^+)$, m_l is the mass of the lepton, and $m_{D_{(s)}^+}$ is the $D_{(s)}^+$ mass. For this pseudoscalar charged particle leptonic decay, the final state neutrino must be left-hand. Due to angular momentum conservation, the final lepton must also be left-hand, since only in this way one obtain a final state with zero angular momentum component in the direction of motion of the leptons. This requirement results in that the decay rate is proportional to m_l^2. In the limit of $m_l = 0$, the D^+ and D_s^+ leptonic decays are forbidden. The leptonic decays can only occur for the case of $m_l \neq 0$. The helicity suppresion of the decay gives a largest decay rate for the final state with the lepton $l = \tau$ and gives a larger decay rate for the final state with the lepton $l = \mu$ than the one for the final state with lepton $l = e$. From the SM, the expected ratios of the decay rates for $\Gamma(D^+ \to \tau^+\nu_\tau)$:$\Gamma(D^+ \to \mu^+\nu_\mu)$:$\Gamma(D^+ \to e^+\nu_e)$ are $2.67 : 1 : 2.4 \times 10^{-5}$, while the expected ratios of the decay rates for $\Gamma(D_s^+ \to \tau^+\nu_\tau)$:$\Gamma(D_s^+ \to \mu^+\nu_\mu)$:$\Gamma(D_s^+ \to e^+\nu_e)$ are $9.8 : 1 : 2.4 \times 10^{-5}$.

In addition to the lowest order decay process, there are some other processes which increase the $D^+(D_s^+)$ leptonic decay rate. These are radiative decay and transition to a virtual $D^{*+}(D_s^{*+})$ by emitting a photon, where $D^{*+}(D_s^{*+})$ decays into $l^+\nu$. The latter transition and decay process is in absence of helicity suppression. These effects should be considered in comparison of the measured decay rates and the expected decay rates with theories based on QCD.

The pseudoscalar decay constants f_{D^+} and $f_{D_s^+}$ are very important constants in heavy flavor physics, which connect to B^+ and B_s^0 mesons decay constants f_{B^+} and $f_{B_s^0}$. As these decay constants are related to the probability of annihilation of the heavy and the light quarks inside the meson, they play an important role both in characterizing the properties of confinement and as absolute normalizations of numerous heavy-flavor transition, including semileptonic decays and non-leptonic decays of the mesons as well as mixing of neutral and anti-neutral meson pairs. For example, the decay constant $f_{B^+(B_s^0)}$ relates to the CKM matrix element $|V_{\rm td(s)}|$ which can be extracted from the $B^0\bar{B}^0$ $(B_s^0\bar{B}_s^0)$ mixing experiment. However, it is currently not possible to precisely measure f_{B^+} from B^+ leptonic decays and is never possible to measure $f_{B_s^0}$ since B_s^0 does not have leptonic decay, so theoretical calculations of f_{B^+} and $f_{B_s^0}$ have to be used in determination of $|V_{\rm td}|$ and $|V_{\rm ts}|$. The decay constants f_{D^+} and $f_{D_s^+}$ as well as f_{B^+} and $f_{B_s^0}$ have been estimated using various theoretical approaches, such as QCD-inspired potential model [3], QCD sum rules [4], lattice QCD [5], and alternative non-perturbative methods [6]. The lattice QCD (LQCD) gives most promising calculations

of these decay constants. The LQCD calculations of the ratios of f_{D^+}/f_{B^+} and $f_{D_s^+}/f_{B_s^0}$ are with higher precision than the calculations of f_{D^+} and f_{B^+} as well as $f_{D_s^+}$ and $f_{B_s^0}$. For this reason, we can use precisely measured f_{D^+} and $f_{D_s^+}$ to valid the LQCD calculations of f_{D^+} and $f_{D_s^+}$. If the LQCD calculations of f_{D^+} and $f_{D_s^+}$ pass the test with the measured f_{D^+} and $f_{D_s^+}$, one can use the calculated ratios of f_{D^+}/f_{B^+} and $f_{D_s^+}/f_{B_s^0}$ combined with the precisely measured f_{D^+} and $f_{D_s^+}$ to obtain f_{B^+} and $f_{B_s^0}$ with high precision or one can use the calculated f_{B^+} and $f_{B_s^0}$ with more confidence to precisely determine the $|V_{\rm td}|$ and $|V_{\rm ts}|$ in $B^0\bar{B}^0$ and $B_s^0\bar{B}_s^0$ mixing experiment, respectively. In addition, with the accurately calculated f_{B^+} one can precisely determine the CKM matrix element $|V_{\rm ub}|$. These improved determinations of $|V_{\rm ub}|$, $|V_{\rm td}|$ and $|V_{\rm ts}|$ would lead to very stringent constraint on the unitary triangle of the CKM matrix.

The CKM matrix elements of $|V_{\rm cd}|$ and $|V_{\rm cs}|$ connect to the leptonic decays of the D^+ and D_s^+ mesons. Historically, measurements of $|V_{\rm cd}|$ were often made based on the measured branching fractions for D meson semileptonic decays and inputs of the form factors for these D meson semileptonic decays or based on the measured neutrino and anti-neutrino interaction. However, due to largely theoretical uncertainties in calculations of the form factor for $D \to \pi l^+ \nu_l$ semilepronic decays, the extracted $|V_{\rm cd}|$ from the measured semileptonic decay branching fractions suffers from an uncertainty as large as 11% [2] and the uncertainty of $|V_{\rm cd}|$ measured from the neutrino and anti-neutrino interaction is as large as 4.8% [2] to date. However, in recent years, the unquenched LQCD calculations of f_{D^+} have reached a high precision of $\sim$ 2% [7]. With the precisely measured branching fraction for $D^+ \to \mu^+\nu_\mu$ decay together with this precisely calculated f_{D^+}, one can more precisely extract $|V_{\rm cd}|$. Similarly, with measured branching fraction of D_s^+ leptonic decay, one can also extract the $|V_{\rm cs}|$.

There could be some possible new physics effects which contribute to the leptonic decays of the D^+ and D_s^+ mesons. Dobrescu and Kronfeld [8], Kundu and Nandi [9] proposed that some non-SM objects participating virtually in the leptonic decays would modify the decay rates observed experimentally. To search for the new physics effects, one needs to carefully compare the measured ratio of $f_{D_s^+}/f_{D^+}$ to the one expected with theories based on QCD. Further more, comparing the values of $|V_{\rm cd}|_{D^+\to l^+\nu}$ and $|V_{\rm cs}|_{D_s^+\to l^+\nu}$ extracted from the leptonic decays of D^+ and D_s^+ mesons to these values of $|V_{\rm cd}|_{\rm CKMfitter}$ and $|V_{\rm cs}|_{\rm CKMfitter}$ determined from the CKMfitter or to the values of $|V_{\rm cd}|_{\rm D\ semileptonic\ decay}$ and $|V_{\rm cs}|_{\rm D\ semileptonic\ decay}$ extracted from D meson semileptonic decays would also provide important information about new physics effects involved in these leptonic decays.

2 Experiments and Methods

The D^+ and D_s^+ mesons can be produced in different kinds of experimental environments, such as e^+e^- annihilation; interaction of hadrons with nuclear targets; interaction of photons or neutrinos with nuclear targets; collision of hadrons. In practice, since the ratio of the signal to the background for leptonic decays of D^+ and D_s^+ mesons is higher at both the e^+e^- collision experiments and fixed target experiments than that at the hadrons collision

experiments, all studies of these leptonic decays are performed at e^+e^- collision experiments and at fixed target experiments.

2.1 e^+e^- collision near threshold

The most clearly experimental environment for studies of these leptonic decays is the e^+e^- experiments operated near open-charm meson pair production energy thresholds. For D^+ leptonic decays, the best center-of-mass energy of the e^+e^- collision is near 3.773 GeV, where the D^+D^- meson pairs are produced. Searching for the leptonic decay of D^+ meson and measurements of leptonic decay branching fractions and decay constant of D^+ meson were made at the historical detectors of MARK-III, BES-I, BES-II and CLEO-c, and today's running BES-III.

Taking the advantage of the D^+D^- production, one first accumulates the samples of the reconstructed D^- mesons in one side, then can absolutely measure the branching fraction for D^+ leptonic decay by examining the decay products in the system recoiling against the D^- tags. In the data analysis, one can search for the $D^+ \to l^+\nu$ in the recoil of the singly tagged D^- mesons by calculating the missing mass square, which is the missing energy square minus the missing momentum square. If there is a neutrino in the recoil side of the tagged D^- meson, the distribution of the missing mass square should characterize with a peak at zero. By examining this missing mass square distribution of the singly tagged D^- mesons together with one charged track which is identified as a lepton, one can fully reconstruct the leptonic decay of $D^+ \to l^+\nu$. Based on the numbers of the fully reconstructed $D^+ \to l^+\nu$ events and the singly tagged D^- mesons, one can well measure the branching fraction for $D^+ \to l^+\nu$ decays, and determine the decay constant f_{D^+}.

Similarly, for D_s^+ leptonic decays, the best center-of-mass energy of the e^+e^- collision is near 4.03 (or 4.17) GeV, where the $D_s^+D_s^-$ ($D_s^+D_s^{*-}$) meson pairs are produced. Historically, the BES-I experiment accumulated data at 4.03 GeV and 4.14 GeV, while the CLEO-c accumulated data at 4.17 GeV, and BES-III accumulated data at 4.01 GeV. The method of measurement of D_s^+ leptonic decay branching fraction and decay constant at these energies are almost the same as these for measurement of D^+ leptonic decay branching fraction and decay constant at 3.773 GeV.

2.2 e^+e^- collision at higher energy

Since the D^+ and D_s^+ mesons can be formed in quark fragmentation, in principle these leptonic decay branching fractions can also be measured by analyzing the data taken at the e^+e^- experiments operated near 10.5 GeV and 91 GeV, where the B factory experiments and Z^0 physics experiments were performed. However, since the D^+ leptonic decays are Cabibbo-suppressed decays, it is difficult to measure the leptonic decay branching fractions and decay constant of D^+ meson with the data collected at these two energies. The data taken at these two energies can be used to measure the leptonic decay branching fractions and decay constant of D_s^+ meson.

The CLEO experiment at the CESR storage ring, BaBar experiment at PEP-II, and

BELLE experiment at asymmetric-energy collider (KEKB) collected or have been collecting large data samples near 10.5 GaV. While ALEPH, L3, and OPAL experiments at the LEP accumulated large data samples of Z^0 hadronic decay events at 91 GeV. With these large data samples of $e^+e^- \to c\bar{c}$ events and $e^+e^- \to Z^0 \to c\bar{c}$ events, the CLEO, BaBar, BELLE, ALEPH, L3, and OPAL experiments measured the branching fractions for $D_s^+ \to l^+\nu$ decays and determined the decay constant $f_{D_s^+}$.

The analysis method used in measurements of the branching fractions for $D_s^+ \to l^+\nu$ with the data taken at 10.5 GeV required the unfolding of the fragmentation process. The total number of D_s^- mesons in the data sample is estimated by reconstructing the four momentum of D_s^- candidates recoiling against the rest of the events. The number of $D_s^+ \to l^+\nu$ events is obtained by identifying a lepton candidate and reconstructing the whole event, including the missing neutrino system.

The analysis method used by ALEPH, L3 and OPAL experiments are based on analysis of fragmentation and decay chain $Z^0 \to c\bar{c}$, $c \to D_s^{*+}$ followed by $D_s^{*+} \to \gamma D_s^+$, $D_s^+ \to \tau^+\nu_\tau$, $\tau^+ \to l^+\nu_e\nu_l$.

2.3 Fixed-target experiments

The fixed-target experiment is other kind of experiment at which the charm mesons can be produced in the interaction of the incident particles with nucleus of the target. From these daughter particles coming from the interaction the D^+ and D_s^+ mesons can be selected and their decay properties can be studied. The cross sections of charm meson production in fixed-target experiments are higher than these at the e^+e^- experiments. However, the non-charm background in the fixed-target experiment are much higher than these at the e^+e^- experiments. To reduce the background events for studying the charm meson decays, the decay length of events are often measured with the vertex detector which is placed near the target. Since the charm mesons have relative long lifetimes, they can travel measurable distances from primary production point. Using the technique of reconstruction of the second vertex of the charm mesons decay, one can well separate the charm meson decay events from light hadron events.

Three fixed-target experiments, the CERN WA75 [18], CERN WA92 [20] and Fermilab E653 [19] studied the D_s^+ leptonic decays. The WA75 [18] experiment is an emulsion-hybrid experiment, which is designed to search for charm quark pair production in 350 GeV/c π^- nucleus interactions. A total of about 80 liters of nuclear emulsion is exposed to a π^- beam from the CERN SPS. The WA92 [20] is designed to study the production and decays of beauty particles from 350 GeV/c π^- interaction in copper and tungsten. Charged particle tracking is performed using the omega spectrometer. The charmed meson decays can also be reconstructed with the spectrometer together with a silicon vertex detector placed near the target. The E653 [19] experiment is also an emulsion-hybrid experiment designed to study production and decays of heavy flavor particles by the direct observation of decay vertex in the emulsion. The charm mesons are from the interaction of a 600 GeV/c π^- and nucleus of the target. These three experiments selected the purely leptonic decays of $D_s^+ \to l^+\nu$ by using transverse momentum spectrum of muons from D_s^+ leptonic decay observed in an

emulsion target.

3 Leptonic decays of D^+ meson

Several experiments performed to search for the leptonic decays of D^+ meson and to precisely measure its leptonic decay branching fractions and decay constant f_{D^+} in the last 25 years. In this section, we first review the available measurements of its leptonic decay branching fractions and decay constant which have been already published, then on behalf of the BES-III collaboration we report new results of precision measurements of the branching fraction for $D^+ \to \mu^+\nu$ decays and decay constant f_{D^+} which are obtained at the BES-III experiment.

3.1 Review of results at old experiments

3.1.1 Search for $D^+ \to l^+\nu$ decay at Mark-III experiment

In 1988, MARK III collaboration first searched for the decay of $D^+ \to l^+\nu$. The MARK-III did not observe any signal events for this decay. They set an upper limit on the decay constant, which is less than 290 MeV at 90% C.L. [10].

3.1.2 First measurements of $B(D^+ \to l^+\nu)$ and f_{D^+} at the BES experiments

In 1998, the BES collaboration analyzed 22.3 pb^{-1} of data taken at 4.03 GeV. From 5 single D tag modes, they found 10082 D^+ mesons produced in their data sample. From this data sample, they found 1 event for $D^+ \to \mu^+\nu$ decay, and measured the branching fraction for $D^+ \to \mu^+\nu$ to be $(0.08^{+0.16+0.05}_{-0.05-0.02})\%$, corresponding to a value of decay constant of $f_{D^+} = (300^{+180+80}_{-150-40})$ MeV [11].

In 2004, the BES collaboration analyzed 33 pb^{-1} of data taken in e^+e^- annihilation with their upgraded BES-II detector at the BEPC collider to study the leptonic decays of D^+ meson. From 9 single D^- tag modes, they accumulated $5321 \pm 149 \pm 160$ D^- tags. In the system recoiling against the D^- tags, they found 3 signal events for $D^+ \to \mu^+\nu$ decays with 0.3 background events estimated with Monte Carlo simulation or estimated with the same data set. With these signal events and the $5321 \pm 149 \pm 160$ D^- tags, they measured the branching fraction for $D^+ \to \mu^+\nu$ decays to be $B(D^+ \to \mu^+\nu) = (0.122^{+0.111}_{-0.053} \pm 0.010)\%$, corresponding to a value of the decay constant $f_{D^+} = (371^{+129}_{-119} \pm 25)$ MeV [12]. These are absolute measurements of the decay branching fraction and decay constant, which do not depend on the yield of D^+ meson production and do not depend on some branching fractions for D^+ meson decay into other modes.

3.1.3 Measurements of $B(D^+ \to l^+\nu)$ and f_{D^+} at CLEO-c experiment

In 2004, the CLEO collaboration analyzed 60 pb^{-1} of data taken in e^+e^- annihilation at 3.770 GeV with the CLEO-c detector at the CESR . From 5 single D^- tag modes, they found $28574 \pm 207 \pm 629$ D^- tags. In the system recoiling against the D^- tags, they found 7 signal events for $D^+ \to \mu^+\nu$ decays, and measured the branching fraction for $D^+ \to \mu^+\nu$

decays to be $B(D^+ \to \mu^+\nu) = (3.5 \pm 1.4 \pm 0.6) \times 10^{-4}$, corresponding to a value of the decay constant $f_{D^+} = (202 \pm 41 \pm 17)$ MeV [13].

In 2005, using 281 pb^{-1} of data taken at 3.770 GeV the CLEO collaboration presented $47.2 \pm 7.1^{+0.3}_{-0.8}$ signal events for $D^+ \to \mu^+\nu$ decay observed in the system recoiling against 158354 ± 496 D^- tags. They measured the decay branching fraction of $B(D^+ \to \mu^+\nu) = (4.40 \pm 0.66^{+0.09}_{-0.12}) \times 10^{-4}$ and extracted the decay constant $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV [14].

In 2008, the CLEO collaboration accumulated 460055 ± 787 D^- tags with 6 hadronic decay modes of the D^- meson from all of 818 pb^{-1} of data taken at 3.773 GeV. They presented 149.7 ± 12.0 signal events for $D^+ \to \mu^+\nu$ decays observed in the system recoiling against these D^- tags. They claimed that they measured the decay branching fraction of $B(D^+ \to \mu^+\nu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$ and determined the decay constant of $f_{D^+} = (205.8 \pm 8.5 \pm 2.5)$ MeV [15]. In measurement of this decay branching fraction and determination of the decay constant, the CLEO assumed that the ratio of the number of the signal events for $D^+ \to \mu^+\nu$ decay over the number of the background events for $D^+ \to \tau^+\nu$ decay in their fitted missing mass squared region is a constant, and they fixed this ratio to the Standard Model value. However, this is not the case of the experimental observation due to that both the number of the events for $D^+ \to \mu^+\nu$ decays and the number of the events for $D^+ \to \tau^+\nu$ decays fluctuate. In addition to these, they assumed that the number of background events do not fluctuate, so they fixed the number of background events in their determination of the number of the signal events. In this case, they obtained the statistical uncertainty in the number of net signal events to be smaller than the square root of the number of the signal events *. In this case, CLEO collaboration reported their measured branching fraction and the decay constant as mentioned above.

However, in the CLEO published paper [15], they also gave conservative results of the decay branching fraction and decay constant, which are $B(D^+ \to \mu^+\nu) = (3.93 \pm 0.35 \pm 0.09) \times 10^{-4}$ and $f_{D^+} = (207.6 \pm 9.3 \pm 2.5)$ MeV. These branching fraction and decay constant were determined in the case of that both the number of events for $D^+ \to \mu^+\nu$ decays and the number of events for $D^+ \to \tau^+\nu$ decays were allowed to be fluctuated in their fit. So these experimental results are more reliable. But these are not appeared in neither the **Abstract** or the **Conclusions** of the CLEO published paper [13]. We will use $B(D^+ \to \mu^+\nu) = (3.93 \pm 0.35 \pm 0.09) \times 10^{-4}$ and $f_{D^+} = (207.6 \pm 9.3 \pm 2.5)$ MeV for our further discussion in this article.

In the system recoiling against 460055 ± 787 D^- tags, the CLEO collaboration found 27.8 ± 16.4 $\tau^+\nu$ with $\tau^+ \to \pi^+\bar{\nu}$ events in their missing mass squired range for the signal. They set an upper limit on the decay branching fraction of $B(D^+ \to \tau^+\nu) < 1.2 \times 10^{-3}$ at 90% C.L..

3.2 New results at BES-III experiment

With the BES-III detector [16] at the BEPC-II [17], the BES-III collaboration collected 2.89 fb^{-1} of data at 3.773 GeV during the time period from 2010 to 2011. With this data sample,

*For example, $\sqrt{149.7} = 12.2$ which is larger than 12.0, where 12.0 is the CLEO reported error of 149.7 signal events observed.

the BES-III made precision measurements of the decay branching fraction for $D^+ \to \mu^+\nu_\mu$ and decay constant f_{D^+}. In this section, we report measurements of the branching fraction for $D^+ \to \mu^+\nu_\mu$ decay and the pseudoscalar decay constant f_{D^+} obtained by analyzing this data sample.

The singly tagged D^- mesons are reconstructed in nine non-leptonic decay modes of $K^+\pi^-\pi^-$, $K^0_s\pi^-$, $K^0_sK^-$, $K^+K^-\pi^-$, $K^+\pi^-\pi^-\pi^0$, $\pi^+\pi^-\pi^-$, $K^0_s\pi^-\pi^0$, $K^+\pi^-\pi^-\pi^-\pi^+$, and $K^0_s\pi^-\pi^-\pi^+$. Events which contain at least three reconstructed charged tracks with good helix fits and their $|\cos\theta| < 0.93$ are selected, where θ is the polar angle of the charged tracks. All tracks, save those from K^0_s decays, must originate from the interaction region, which require that the closest approach of a charged track in the xy plane is less than 1.0 cm and is less than 15.0 cm in the z direction. Pions and kaons are identified by means of TOF and dE/dx measurements with which the combined confidence levels CL_π and CL_K for pion and kaon hypotheses are, respectively, calculated. Pion (kaon) identification requires $CL_\pi > CL_K$ ($CL_K > CL_\pi$) for its momentum $p < 0.75$ GeV/c and $CL_\pi > 0.1\%$ ($CL_K > 0.1\%$) for its momentum $p \geq 0.75$ GeV/c.

To select good photons from the π^0 meson decays, the energy of the photon deposited in the barrel (end-cap) EMC is required to be greater than 0.025 (0.050) GeV. The barrel (end-cap) EMC covers the range of $|\cos\theta_\gamma| < 0.83$ ($0.85 \leq |\cos\theta_\gamma| < 0.93$), where θ_γ is the polar angle of the photon. In addition, the EMC cluster timing TDC is required to be in the range of $0 \leq$ TDC ≤ 700 ns. In order to reduce background the angle between the photon and the nearest charged track is required to be greater than $10°$. To further reduce the combinatorial background, the 1-C kinematic fit is performed to constrain the invariant mass of $\gamma\gamma$ to the mass of π^0 meson. If the 1-C kinematic fit is successful these $\gamma\gamma$ are kept as good candidates for $\pi^0 \to \gamma\gamma$ decay.

To select K^0_s decays, a second vertex fit is subjected to two charged tracks with opposite charge and the χ^2 from the vertex fit is required to be less than 999.0. In addition, the secondary vertex from which the $\pi^+\pi^-$ pair originate should be displaced from the event vertex at least by the decay length $L_{xyz} > 0$ mm. After these, only the $\pi^+\pi^-$ meson pair with invariant mass $M_{\pi^+\pi^-}$ being within about $\pm 3.5\sigma$ mass window of the nominal K^0_s mass is taken as the K^0_s meson candidate.

The singly tagged D^- mesons are fully reconstructed by requiring the difference in the energy, ΔE, of the daughter particle $mKn\pi$ (where m=0, 1, 2; n = 0, 1, 2, 3, or 4) system with the beam energy. They then require $|\Delta E| < (2 \sim 3)\sigma_{E_{mKn\pi}}$, where $\sigma_{E_{mKn\pi}}$ is the standard deviation of the distribution of the energy of $mKn\pi$ system, and then examine the beam energy constraint mass of the tagged $mKn\pi$ system,

$$M_{\rm B} = \sqrt{E^2_{\rm beam} - |\vec{p}_{mKn\pi}|^2}, \tag{2}$$

where $E_{\rm beam}$ is the beam energy, and $|\vec{p}_{mKn\pi}|$ is the magnitude of the momentum of the daughter particle $mKn\pi$ system.

The $M_{\rm B}$ distributions for the nine D^- tag modes are shown in Fig. 2. A maximum likelihood fit to the mass spectrum with a Crystal Ball function plus an Gaussian function for the D^- signal and the ARGUS function to describe background yields the number of the singly tagged D^- events for each of the nine modes. Selecting these candidates for D^-

tags within the range marked by arrows in Fig. 2 reduce signal number by about 2% giving a total of 1586056 ± 2327 D^- tags. In these D^- tags, 20103 D^- tags are reconstructed in more than one single D^- tag mode. Subtracting this number of the double counting D^- tags from the 1586056 ± 2327 D^- tags yields 1565953 ± 2327 D^- tags which are used for further analysis of measuring the branching fraction for $D^+ \to \mu^+\nu_\mu$ decays.

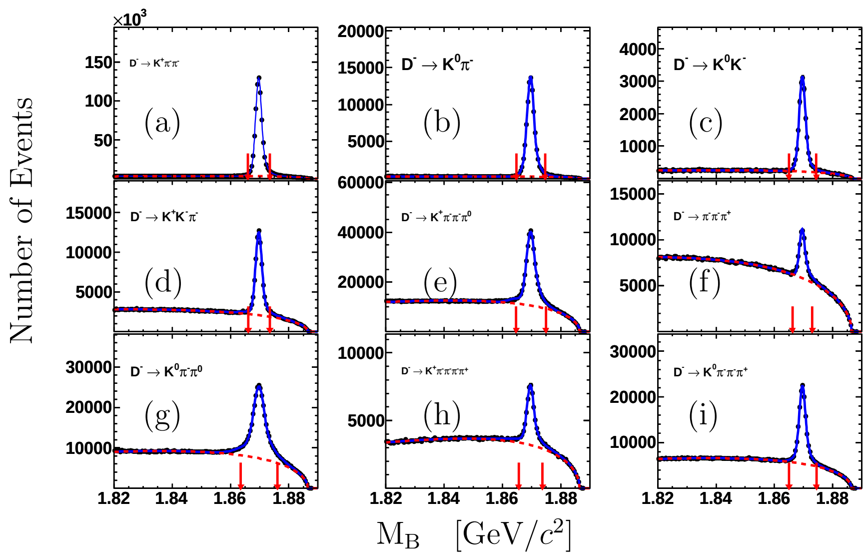

Figure 2: Distributions of the beam energy constraint masses of the $mKn\pi$ combinations for the 9 single tag modes from the data; where (a), (b), (c), (d), (e), (f), (g), (h), (i) are for the modes of $D^- \to K^+\pi^-\pi^-$, $D^- \to K^0_s\pi^-$, $D^- \to K^0_sK^-$, $D^- \to K^+K^-\pi^-$, $D^- \to K^+\pi^-\pi^-\pi^0$, $D^- \to \pi^+\pi^-\pi^-$, $D^- \to K^0_s\pi^-\pi^0$, $D^- \to K^+\pi^-\pi^-\pi^-\pi^+$, and $D^- \to K^0_s\pi^-\pi^-\pi^+$, respectively.

Candidate events for the decay $D^+ \to \mu^+\nu_\mu$ are selected from the surviving charged tracks in the system recoiling against the singly tagged D^- mesons. To select the $D^+ \to \mu^+\nu_\mu$, it is required that there be a single charged track originating from the interaction region in the system recoiling against the D^- tag and the charged track satisfies $|\cos\theta| < 0.93$ as well as it is identified as a μ^+. The μ^+ can be well identified with the passage length of a charged particle through the MUC since a charged hadron (pion or kaon) quickly loses its energy due to its strong interactions with the absorber of the MUC and most of the hadrons stop in the absorber before passing a long passage length in the MUC. For the candidate event, no extra good photon which is not used in the reconstruction of the singly tagged D^- meson is allowed to be present in the event, where the "good photon" is the one with deposited energy in the EMC being greater than 300 MeV.

Since there is a missing neutrino in the purely leptonic decay event, the event should be characteristic with missing energy E_{miss} and missing momentum p_{miss} which are carried away by the neutrino. So they infer the existence of the neutrino by requiring a measured value of the missing mass squared $M^2_{\rm miss}$ to be around zero. The missing mass squared $M^2_{\rm miss}$ is defined as

$$M^2_{\rm miss} = (E_{\rm beam} - E_{\mu^+})^2 - (-\vec{p}_{D^-_{\rm tag}} - \vec{p}_{\mu^+})^2, \qquad (3)$$

where E_{μ^+} and $\vec{p}_{\mu^+}$ are, respectively, the energy and three-momentum of the μ^+, and $\vec{p}_{D^-_{\rm tag}}$ is three-momentum of the candidate for D^- tag.

Figure 3(a) and (b) show the scatter-plots of the momentum of the identified muon satisfying the requirement for selecting $D^+ \to \mu^+\nu_\mu$ decay versus M^2_{miss}, where the blue box in Fig. 3(a) shows the signal region for $D^+ \to \mu^+\nu_\mu$ decays. Within the signal region, there are 425 candidate events for $D^+ \to \mu^+\nu_\mu$ decay. The two concentrated clusters out side of the signal region are from D^+ non-leptonic decays and some other background events. The events whose peak is around 0.25 GeV$^2/c^4$ in M^2_{miss} are mainly from $D^+ \to K^0_L\pi^+$ decays, where K^0_L is missing. Projecting the events for which the identified muon momentum being in the range from 0.8 to 1.1 GeV/c onto the horizontal scale yields the M^2_{miss} distribution as shown in Fig. 3(c), where the difficultly suppressed backgrounds from $D^+ \to K^0_L\pi^+$ decays in CLEO-c measurement [15] are effectively suppressed due to that they use the MUC measurements to identify the muon.

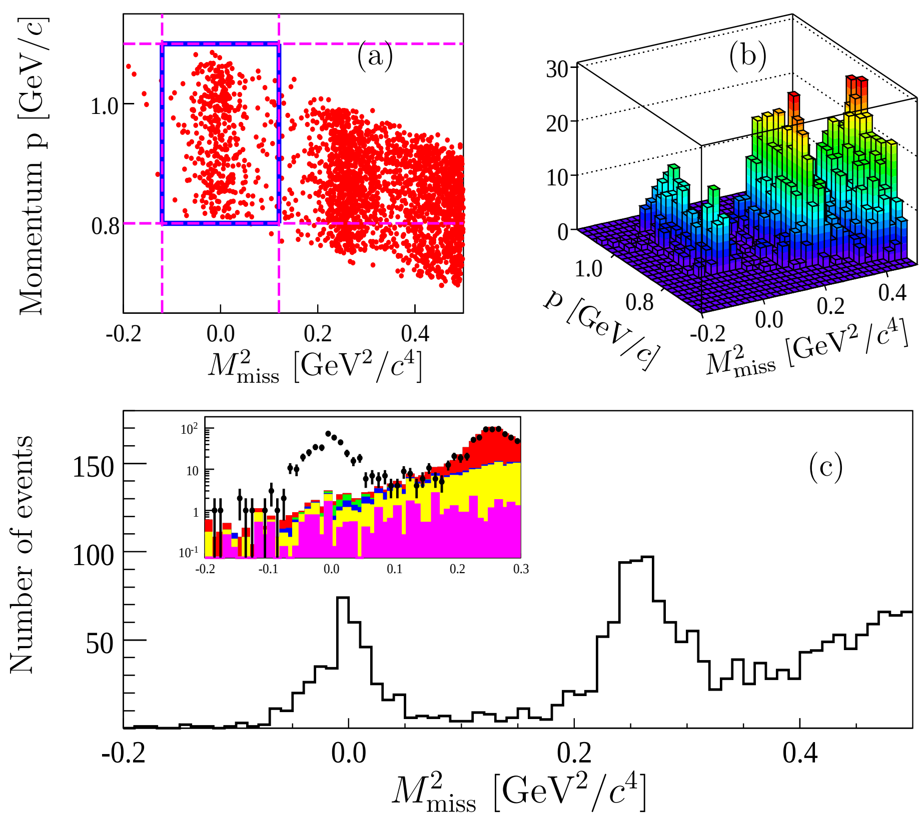

Figure 3: Distributions of $M^2_{\rm miss}$, where (a) and (b) are scatter plots of the identified muon momentum p VS $M^2_{\rm miss}$, and (c) is the distribution of $M^2_{\rm miss}$. The insert shows the signal region for $D^+ \to \mu^+\nu_\mu$ on a log scale, where dots with error bars are for the data, histograms are for the simulated backgrounds from $D^+ \to K^0_L\pi^+$ (red), $D^+ \to \pi^0\pi^+$ (green), $D^+ \to \tau^+\nu_\tau$ (blue) and other decays of D mesons (yellow) as well as from $e^+e^- \to$non-$D\bar{D}$ decays (pink).

Some non-purely leptonic decay events from the D^+, D^0, $\gamma\psi(3686)$, $\gamma J/\psi$, $\psi(3770) \to$ non $- D\bar{D}$, $\tau^+\tau^-$ decays as well as continuum light hadron production may also satisfy the selection criteria and are the background events to the purely leptonic decay events. These

Table 1: Sources of background events for $D^+ \to \mu^+\nu_\mu$.

Source mode	Number of events
$D^+ \to K_L^0\pi^+$	7.9 ± 0.8
$D^+ \to \pi^+\pi^0$	3.8 ± 0.5
$D^+ \to \tau^+\nu_\tau$	6.9 ± 0.7
Other decays of D mesons	17.9 ± 1.1
$e^+e^- \to \gamma\psi(3686)$	0.2 ± 0.2
$e^+e^- \to \gamma J/\psi$	0.0 ± 0.0
$e^+e^- \to$ *light hadron* (continuum)	8.2 ± 1.4
$e^+e^- \to \tau^+\tau^-$	1.9 ± 0.5
$\psi(3770) \to non - D\bar{D}$	0.9 ± 0.4
Total	47.7 ± 2.3

Table 2: Sources of the relative systematic uncertainties in the measured branching fraction for $D^+ \to \mu^+\nu_\mu$ decay.

Source	Systematic uncertainty [%]
Number of D^- tags ($\mathrm{N}_{D^-_{tag}}$)	0.6
Muon tracking	0.5
μ selection	0.3
$\mathrm{E}_{\gamma_{\max}}$ cut	0.7
Muon momentum cut	0.1
M^2_{miss} cut	0.5
Background estimation	0.7
Monte Carlo statistics	0.2
Radiative correction	1.0
Total	1.7

background events must be subtracted off. The number of the background events can be estimated by analyzing different kinds of Monte Carlo simulation events. Detailed Monte Carlo studies show that there are $47.7 \pm 2.3 \pm 1.3$ background events in 425 candidates for $D^+ \to \mu^+\nu_\mu$ decays, where the first error is due to Monte Carlo statistic and second systematic arising from uncertainties in the branching fractions or production cross sections for the source modes as shown in Table 1.

After subtracting the number of background events, $377.3 \pm 20.6 \pm 2.6$ signal events for $D^+ \to \mu^+\nu_\mu$ decay are retained, where the first error is statistical and the second systematic arising from the uncertainty of the background estimation.

The overall efficiency for observing the decay $D^+ \to \mu^+\nu_\mu$ is obtained by analyzing full Monte Carlo simulation events of $D^+ \to \mu^+\nu_\mu$ VS D^- tags and combining with μ^+ reconstruction efficiency in the MUC. The μ^+ reconstruction efficiency in the MUC is measured with muon samples selected from the same data taken at 3.773 GeV. The overall efficiency is $0.6382 \ \pm 0.0015$.

With 1565953 singly tagged D^- mesons, $377.3 \pm 20.6 \pm 2.6$ $D^+ \to \mu^+\nu_\mu$ decay events observed and the efficiency 0.6382 ± 0.0015, the BES-III collaboration obtain the branching fraction

$$B(D^+ \to \mu^+\nu_\mu) = (3.74 \pm 0.21 \pm 0.06) \times 10^{-4} \quad \text{(BESIII Preliminary)},$$

where the first error is statistical and the second systematic. The sources of the systematic uncertainties are summarized in Table 2. This measured branching fraction is consistent within error with world average of $B(D^+ \to \mu^+\nu_\mu) = (3.82 \pm 0.33) \times 10^{-4}$ [2], but with more precision.

The decay constant f_{D^+} can be obtained by inserting the measured branching fraction, the mass of the muon, the mass of the D^+ meson, the CKM matrix element $|V_{\rm cd}| = 0.2252 \pm 0.0007$ from the CKMFitter [2] G_F and the lifetime of the D^+ meson [2] into Eq.(1), which yields

$$f_{D^+} = (203.91 \pm 5.72 \pm 1.97)\ \text{MeV} \quad \text{(BESIII Preliminary)},$$

where the first errors are statistical and the second systematic arising mainly from the uncertainties in the measured branching fraction (1.7%), the CKM matrix element $|V_{\rm cd}|$ (0.3%), and the lifetime of the D^+ meson (0.7%) [2]. The total systematic error is 1.0%.

4 Leptonic decays of D_s^+ meson

The first observation of D_s^+ leptonic decay is performed at the WA75 [18] fixed-target experiment in 1992. Since then experimental studies of the D_s^+ leptonic decays have been performed at e^+e^- experiments operated near $D_s^+D_s^-$ ($D_s^+D_s^{*-}$) production threshold, at energies of the peak of $\Upsilon(2S)$ production and Z^0 production in e^+e^- annihilation, as well as at other fixed-target experiments. In this section, we review all of these available measurements of the D_s^+ leptonic decay branching fractions and decay constants $f_{D_s^+}$.

4.1 Results at fixed-target experiment

In 1992, the WA75 [18] collaboration reported the first measurement of the branching fraction for $D_s^+ \to \mu^+\nu$ decay and measurement of the decay constant $f_{D_s^+}$. To search for $D_s^+ \to \mu^+\nu$ decay events they examined distribution of the muon momentum p_t^μ perpendicular to the direction of flight of the charm mesons. Figure 4 (a) shows this momentum distribution for candidates consistent with the decay of a charged particle decaying to a single charged particle, while Fig. 4 (b) shows this momentum distribution for candidates consistent with the decay of a neutral particle decaying to two charged particles. The kinematic upper limit on p_t^μ is 0.98 GeV/c for $D_s^+ \to \mu^+\nu$ and 0.93 GeV/c for $D^+ \to \mu^+\nu$, while the kinematic upper limit on p_t^μ is 0.88 GeV/c for semileptonic decays. With these different kinematic signatures of p_t^μ distributions at high transverse momentum region, the leptonic decay of $D_s^+ \to \mu^+\nu$ events can be well separated from other background events of charm decays. By comparing the Fig. 4 (a) and Fig. 4 (b), one can find that, in the charged topology, six events are observed with $p_t^\mu > 0.9$ GeV/c, while no event is observed above $p_t^\mu > 0.9$ GeV/c in the neutral topology.

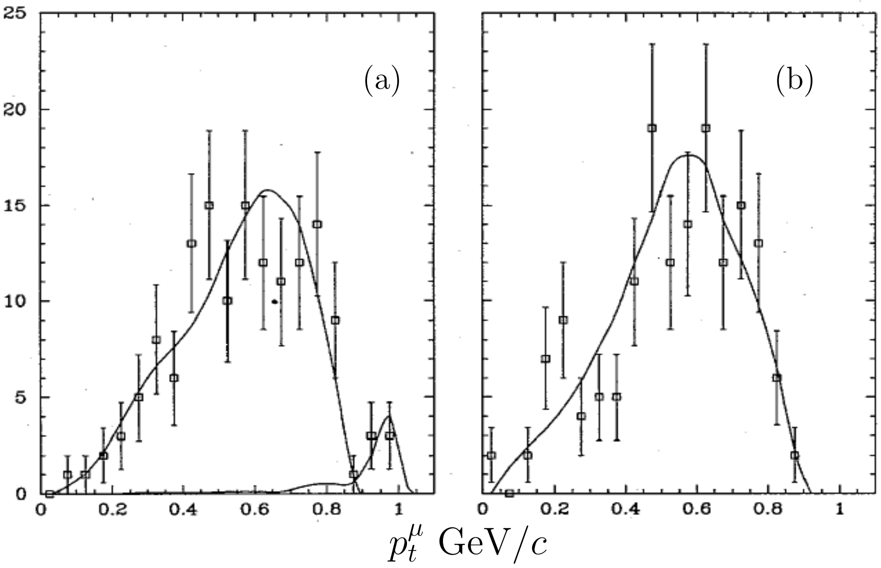

Figure 4: Distribution of muon momentum perpendicular to the direction of flight of the charm mesons observed at the WA75 [18] experiment, where (a) is for the candidates consistent with the decay of the charm meson to a single charged particle, and (b) is for the candidates consistent with the decay of the charm meson to two charged particles. The lines show the Monte Carlo predictions for these decays.

The estimated number of the background events from $D^+ \to \mu^+\nu$ decay is 0.6 ± 0.2 events. Based on these six candidate events for $D_s^+ \to \mu^+\nu$ decay, 0.6 ± 0.2 background events from $D^+ \to \mu^+\nu$ and the number of events of $D^0 \to \mu^+\nu X$ for normalization, the WA75 [18] collaboration determined a branching fraction of $B(D_s^+ \to \mu^+\nu) = (4.0^{+1.8+0.8}_{-1.4-0.6} \pm 1.7) \times 10^{-3}$, and a decay constant of $f_{D_s^+} = (225 \pm 45 \pm 20 \pm 40)$ MeV [18].

As WA75 experiment, the Fermilab E635 [19] is a fixed-target experiment with an emulsion target and muon trigger. In 1996, the E653 [19] collaboration observed 23 events for $D_s^+ \to \mu^+\nu$ leptonic decays in the fixed-target experiment. Based on the yields of $D_s^+ \to \phi\mu^+\nu$ signal observed in the same data sample, the E653 [19] collaboration determined a relative decay branching fraction and decay constant of $B(D_s^+ \to \mu^+\nu)/B(D_s^+ \to \phi\mu^+\nu) = (0.16 \pm 0.06 \pm 0.03)$ and $f_{D_s^+} = (194 \pm 35 \pm 20 \pm 14)$ MeV [19], respectively.

In 2000, using almost the same analysis technique as the one used by WA75 [18], the BEATRICE collaboration observed $D_s^+ \to \mu^+\nu$ leptonic decays at the WA92 experiment. They measured a relative decay branching fraction and decay constant of $B(D_s^+ \to \mu^+\nu)/B(D_s^+ \to \phi_{(K^+K^-)}\pi^+) = 0.47 \pm 0.13 \pm 0.04 \pm 0.06$ and $f_{D_s^+} = (323 \pm 44 \pm 12 \pm 34)$ MeV [20], respectively.

4.2 Results at e^+e^- experiments near $D_s^+D_s^-$ thresholds

4.2.1 BES-I experiment near $D_s^+D_s^-$ threshold

In 1995, by analyzing the data taken at 4.03 GeV with the BES-I detector operated at the BEPC collider, the BES collaboration reconstructed 94.3 ± 12.5 singly tagged D_s^- mesons with three hadronic decay modes. In the system recoiling against the singly tagged D_s^-

mesons, the BES collaboration found 3 events of both the $D_s^+ \to \tau^+\nu$ and $D_s^+ \to \mu^+\nu$. They measured the decay branching fractions for $D_s^+ \to \tau^+\nu$ and $D_s^+ \to \mu^+\nu$ to be $B(D_s^+ \to \tau^+\nu) = (15^{+13+3}_{-6-2})\%$ and $B(D_s^+ \to \mu^+\nu) = (1.5^{+1.3+0.3}_{-0.6-0.2})\%$, respectively. They extracted a value of the decay constant $f_{D_s^+} = (430^{+150}_{-130} \pm 40)$ MeV [21], where the first error is statistical and the second is the systematic uncertainty arising from the uncertainties of reconstruction efficiency, background estimation and the D_s^+ lifetime.

These are the first absolute measurements of these decay branching fractions and decay constant, which do not need to normalize to other D_s^+ decay modes and do not depend on the knowing D_s^+ production rate in the data samples.

4.2.2 CLEO-c experiment near $D_s^+ D_s^{*-}$ threshold

In 2009, the CLEO-c analyzed 600 pb^{-1} of data taken at 4.17 GeV in e^+e^- annihilation to measure branching fractions for $D_s^+ \to l^+\nu$ decays and decay constant $f_{D_s^+}$. From this data sample, they accumulated the singly tagged D_s^- mesons using 9 hadronic decay modes. Since the $D_s^+ D_s^{*-}$meson pairs are produced in e^+e^- collision, the CLEO collaboration used the missing mass square method to reconstruct the decay of $D_s^{*-} \to \gamma D_s^-$. They calculate the variable $MM^{*2} = (E_{\rm CM} - E_{D_s^-} - E_\gamma)^2 - (p_{\rm CM} - p_{D_s^-} - p_\gamma)^2$ for each event. MM^{*2} is the missing mass-squared of the system recoiling against the D_s^{*-}. With the peak of MM^{*2} distributions for each of nine decay modes, they clearly reconstructed the D_s^+ mesons. By fitting to these MM^{*2} distributions, they obtained the number of the D_s^+ mesons in total. In the system recoiling against the singly tagged D_s^{*-}, they searched for the leptonic decays of $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$. Finally, they measured the decay branching fractions of $B(D_s^+ \to \mu^+\nu) = (0.565 \pm 0.045 \pm 0.017)\%$ and $B(D_s^+ \to \tau^+\nu) = (6.42 \pm 0.81 \pm 0.18)\%$, and extracted the decay constant of $f_{D_s^+} = (263.3 \pm 8.2 \pm 3.9)$ MeV [22].

Using three cleanest singly tagged D_s^- hadronic decay modes to accumulate the D_s^- tags, the CLEO collaboration searched for $D_s^+ \to \tau^+\nu \to e^+\nu\nu\nu$ decays. They measured the decay branching fraction and decay constant of $B(D_s^+ \to \tau^+\nu) = (5.30 \pm 0.47 \pm 0.22)\%$ and $f_{D_s^+} = (252.5 \pm 11.1 \pm 5.2)$ MeV [23], respectively.

In addition to the $D_s^+ \to \tau^+\nu \to e^+\nu\nu\nu$ decay mode, the CLEO also observed $D_s^+ \to \tau^+\nu \to \rho^+\nu\nu$ decays. With this decay process the CLEO measured the $D^+ \to \tau^+\nu$ decay branching fraction, which is $B(D_s^+ \to \tau^+\nu) = (5.52 \pm 0.57 \pm 0.21)\%$. With this decay branching fraction, they extracted the decay constant of $f_{D_s^+} = (257.8 \pm 13.3 \pm 5.2)$ MeV [24].

4.3 Results at e^+e^- experiments operated at higher energies

As we mentioned before, the D_s^+ meson can also be produced from the quark fragmentation process in continuum $c\bar{c}$ production as well as produced in Z^0 decays. So the D_s^+ leptonic decays can be studied with the large data samples taken at $\Upsilon(4S)$ production energy and taken at Z^0 production energy. In this section we review the results on measurements of branching fractions of D_s^+ leptonic decays and decay constant $f_{D_s^+}$ measured at these two energies.

4.3.1 Results at e^+e^- experiments operated at 10.5 GeV

Several e^+e^- experiments operated at 10.5 GeV have studied or have been studying the D_s^+ leptonic decays.

In 1998, the CLEO-II observed 182 ± 22 events for $D_s^{*+} \to \gamma D_s^+$ followed by $D_s^+ \to \mu^+\nu$ by analyzing using 5 million $e^+e^- \to c\bar{c}$ events. They measured the decay width ratio of $\Gamma(D_s^+ \to \mu^+\nu)/\Gamma(D_s^+ \to \phi\pi^+) = 0.173 \pm 0.023 \pm 0.035$ and determined the decay constant of $f_{D_s^+} = (280 \pm 19 \pm 28 \pm 34)$ MeV [25].

In 2008, BELLE collaboration made measurements of leptonic decay branching fractions of D_s^+ meson. They selected the D_s^+ leptonic decays from the $e^+e^- \to c\bar{c}$ continuum production, during which the $D_s^* D^{\pm,0} K^{\pm,0} X$ produced from the quark fragmentation, where $D_s^* \to \gamma D_s$ and X indicates several pions or photons. By reconstructing the recoil mass of the $DKX\gamma$, they observed clear D_s signal in the system recoiling against the $DKX\gamma$. By fitting the mass distributions of the system recoiling against $DKX\gamma$, they accumulated $32100 \pm 870 \pm 1210$ D_s events. Then they examined the mass distribution of the system recoiling against the $DKX\gamma\mu$ combinations. They found a very clear signal for $D_s^+ \to l^+\nu$ decays with a narrow peak around 0.0 in the missing mass squared $M^2_{\rm REC}(DKX\gamma\mu)$ distribution. Fitting this $M^2_{\rm REC}(DKX\gamma\mu)$ distribution yields $169 \pm 16 \pm 8$ signal events for $D_s^+ \to l^+\nu$ decays. With these numbers, the BELLE collaboration measured the decay branching fraction of $B(D_s^+ \to \mu^+\nu) = (0.644 \pm 0.076 \pm 0.057)\%$, and decay constant of $f_{D_s^+} = (275 \pm 16 \pm 12)$ MeV [26].

At the Charm2012 Conference, the BELLE collaboration presented an updated analysis of their 913 fb^{-1} of data collected at 10.6 GeV. With a larger data sample, the BELLE collaboration observed 489 ± 26 signal events for $D_s^+ \to \mu^+\nu$ decay and measured the decay branching fraction of $B(D_s^+ \to \mu^+\nu) = (0.528 \pm 0.028 \pm 0.019)\%$. In addition to this decay, the BELLE collaboration observed 2206 ± 84 signal events for $D_s^+ \to \tau^+\nu$ with the decays of $\tau^+ \to e^+\nu\nu$, $\tau^+ \to \mu^+\nu\nu$ and $\tau^+ \to \pi^+\nu$, and they measured the decay branching fraction of $B(D_s^+ \to \tau^+\nu) = (5.70 \pm 0.21^{+0.31}_{-0.30})\%$. With these two decay modes together, they extracted the decay constant of $f_{D_s^+} = (255.0 \pm 4.2 \pm 5.0)$ MeV [27].

In 2010, using the same technique as the one used by the BELLE collaboration, the BaBar collaboration made measurements of the D_s leptonic decay branching fractions and determined decay constant. By analyzing 521 fb^{-1} of data taken at 10.6 GeV, the BaBar collaboration measured the decay branching fractions for $D_s^+ \to \mu^+\nu$, $D_s^+ \to \tau^+\nu$ ($\tau^+ \to e^+\nu\nu$), and $D_s^+ \to \tau^+\nu$ ($\tau^+ \to \mu^+\nu\nu$) to be $B(D_s^+ \to \mu^+\nu) = (0.602 \pm 0.038 \pm 0.034)\%$, $B(D_s^+ \to \tau^+\nu) = (5.07 \pm 0.52 \pm 0.68)\%$ and $B(D_s^+ \to \tau^+\nu) = (4.91 \pm 0.47 \pm 0.54)\%$, respectively, and determined the decay constant of $f_{D_s^+} = (258.6 \pm 6.4 \pm 7.5)$ MeV [28].

4.3.2 Results at e^+e^- experiments operated at 91 GeV

In e^+e^- annihilation at 91 GeV, the Z^0 bosons are produced. The Z^0 boson can decay into $c\bar{c}$. Due to quark fragmentation the D_s^+ meson are produced in the final states of the Z^0 decays. At these experiments, the decays of $D_s^+ \to l^+\nu$ are selected by reconstructing the decay sequence of $e^+e^- \to Z^0 \to c\bar{c} \to D_s^{*-}X$, where $D_s^{*-} \to \gamma D_s^-$ with $D_s^- \to l^-\nu$.

In 1997, L3 collaboration observed 15.5 ± 6.0 $D_s^+ \to \tau^+\nu$ events coming from 1.5×10^6

$Z^0 \to q\bar{q}(\gamma)$ events. They measured the leptonic decay branching fraction of $B(D_s^+ \to \tau^+\nu) = (7.4 \pm 2.8 \pm 1.6 \pm 1.8)\%$, and decay constant of $f_{D_s^+} = (309 \pm 58 \pm 33 \pm 38)$ MeV [29].

In 2001, the OPAL collaboration observed 22.5 ± 6.9 $D_s^+ \to \tau^+\nu$ events coming from 3.9×10^6 $Z^0 \to q\bar{q}(\gamma)$ events. They measured the leptonic decay branching fraction of $B(D_s^+ \to \tau^+\nu) = (7.0 \pm 2.1 \pm 2.0)\%$, and decay constant of $f_{D_s^+} = (286 \pm 44 \pm 41)$ MeV [30].

In 2002, the ALEPH collaboration made a measurement of leptonic decay branching fractions and decay constant of D_s^+ meson. The measurements were made based on an almost same technique as the one used by the L3 and OPAL collaborations. But the ALEPH reconstructed the $D_s^+ \to \mu^+\nu$ decay directly. By analyzing 3.97×10^6 hadronic Z^0 decays, they measured the leptonic decay branching fractions of $B(D_s^+ \to \tau^+\nu) = (5.79 \pm 0.77 \pm 1.84)\%$ and $B(D_s^+ \to \mu^+\nu) = (0.68 \pm 0.11 \pm 0.18)\%$, and decay constant of $f_{D_s^+} = (285 \pm 19 \pm 40)$ MeV [31].

5 Comparison of measured and expected decay constants of f_{D^+} and $f_{D_s^+}$

5.1 Re-determine $f_{D^+_{(s)}}$

The values of the decay constants of f_{D^+} and $f_{D_s^+}$ measured at different experiments were historically obtained with the measured leptonic decay branching fractions at these experiments, with the lifetimes and masses of the D^+ and D_s^+ mesons, together with the CKM matrix elements of $|V_{\rm cd}|$ and $|V_{\rm cs}|$, or with the measured branching fractions for $D_s^+ \to \phi\pi^+$ decay or other decays as inputs. The historical values of the lifetimes, the CKM matrix elements and branching fractions for $D_s^+ \to \phi\pi^+$ decay or other decays used in determination of the values of the f_{D^+} and $f_{D_s^+}$ differ from each at these experiments. In order to make precise comparison of these measured decay constants, we re-calculate the decay constants based on the originally measured branching fractions for these two leptonic decays of the D^+ and D_s^+ mesons. In re-determination of the decay constants f_{D^+} and $f_{D_s^+}$, the values of physical quantities used are listed in Table 3, which are quoted from PDG2010 [2].

Table 3: The values of physical quantities used in the re-determination of f_{D^+} and $f_{D_s^+}$.

$D^+_{(s)}$ mass	$D^+_{(s)}$ lifetime	lepton mass	$\lvert V_{cd}\rvert$ or $\lvert V_{cs}\rvert$
$m_{D^+} = (1869.60 \pm 0.16)$ MeV	$\tau_{D^+} = (1040 \pm 7) \times 10^{-15}$ s	$m_\mu = (105.658367 \pm 0.000004)$ MeV	$\lvert V_{cd}\rvert = 0.2252 \pm 0.0007$
$m_{D_s^+} = (1968.47 \pm 0.33)$ MeV	$\tau_{D_s^+} = (500 \pm 7) \times 10^{-15}$ s	$m_\tau = (1776.82 \pm 0.16)$ MeV	$\lvert V_{cs}\rvert = 0.97345^{+0.00015}_{-0.00016}$

5.2 Comparison of the measured and expected $f_{D^+_{(s)}}$

Decay constants for pseudoscalar mesons containing a heavy c and/or b quark have been predicted with theories or models based on the QCD. In recent years, the LQCD calculations of the decay constants $f_{D^+_{(s)}}$ have achieved high precision. Some theoretical predictions for the decay constants were calculated in Refs. [7, 41].

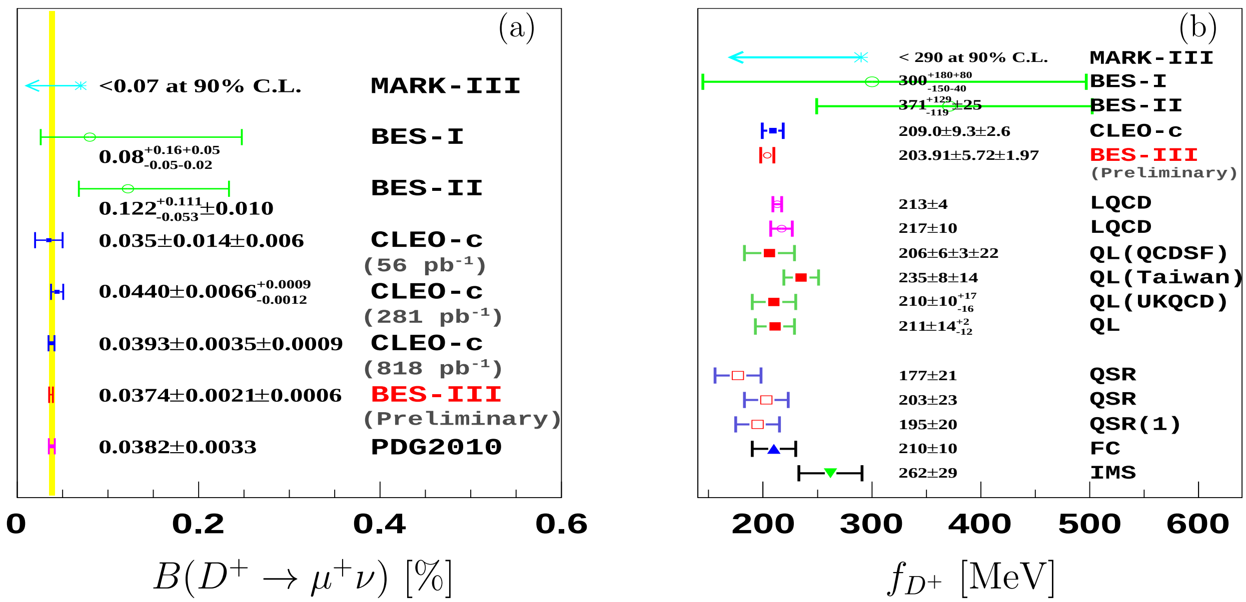

Figure 5: (a) Comparison of the measured branching fractions for $D^+ \to \mu^+\nu$ decay, and (b) comparison of the measured decay constant f_{D^+} and these predicted with theories [7, 41], where QSR is for QCD Sum Rule, FC is for Field Correlations, and IMS is for Isospin Mass Splittings.

Figure 5 (a) and (b) give comparison of the measured branching fractions for $D^+ \to \mu^+\nu$ and comparison of the measured values of the decay constant f_{D^+} with those predicted with different theoretical calculations, respectively. The weighted average of the predicted values of decay constant f_{D^+} with theories based on QCD is $f_{D^+} = (212.7 \pm 3.2)$ MeV †.

Figure 6 (a) and (b) show comparison of the branching fractions for $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ decays measured at different experiments, respectively. With these measured decay branching fractions, we obtain the average branching fraction for $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ decays to be $B(D_s^+ \to \mu^+\nu) = (0.553 \pm 0.024)\%$ and $B(D_s^+ \to \tau^+\nu) = (5.56 \pm 0.23)\%$, respectively.

With these branching fractions for the two leptonic decays measured at the different experiments, we obtain the values of the decay constant $f_{D_s^+}$. Figure 7 (a) and (b) show comparison of the measured values of $f_{D_s^+}$ obtained from the decays of $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$, respectively. Weighting these measured values of $f_{D_s^+}$ with these errors yields the decay constant of $f_{D_s^+} = (253.8 \pm 6.3)$ MeV and $f_{D_s^+} = (259.1 \pm 5.5)$ MeV obtained from the decays of $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$, respectively. Figure 8 (a) shows a comparison of these two averaged decay constant $f_{D_s^+}$ obtained from $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ decays and the weighted average of the two decay constants. Figure 8 (b) shows comparison of values of $f_{D_s^+}$ predicted with theories. Weighted the values of $f_{D_s^+}$ predicted with theories based on QCD with their errors yields an average value of the theoretically predicted decay

†We did not use the predicted values given by QSR(1) and IMS in calculating the weighted average of the predicted values of decay constant f_{D^+} since the ratios of $f_{D_s^+}/f_{D^+}$ are not available in Refs.

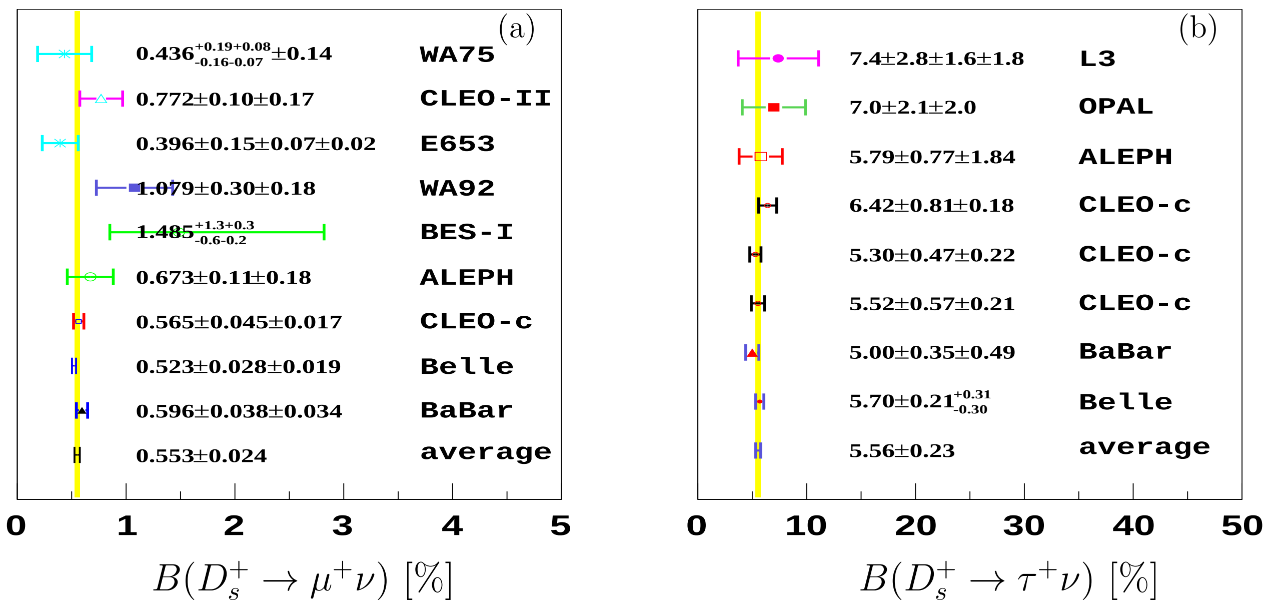

Figure 6: Comparison of the measured branching fractions for (a) $D_s^+ \to \mu^+\nu$ decay and (b) $D_s^+ \to \tau^+\nu$ decay.

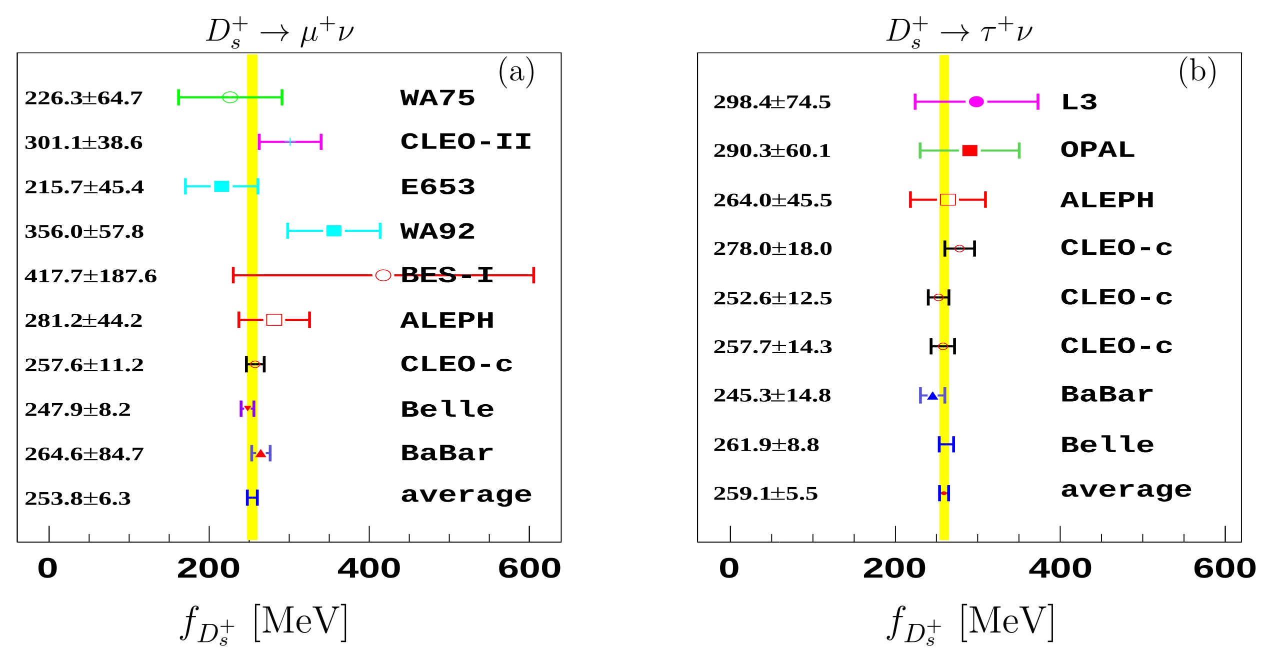

Figure 7: Comparison of the measured $f_{D_s^+}$ obtained from (a) $D_s^+ \to \mu^+\nu$ decay and (b) $D_s^+ \to \tau^+\nu$ decay.

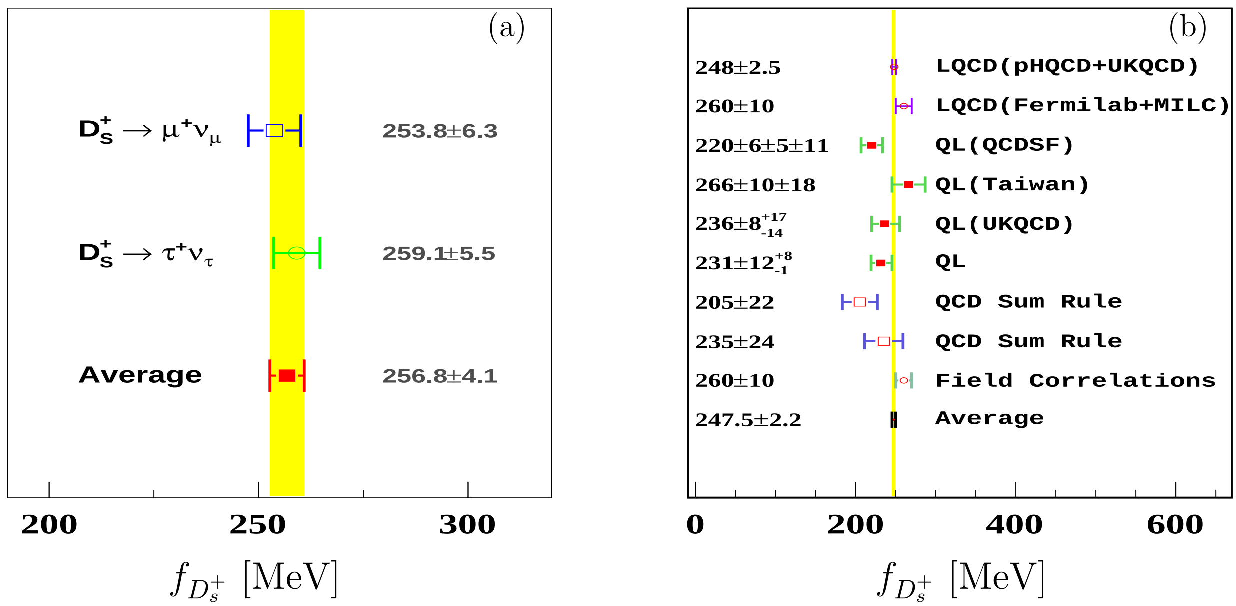

Figure 8: Comparison of (a) the measured $f_{D_s^+}$ obtained from $D_s^+ \to \mu^+\nu$ decay and $D_s^+ \to \tau^+\nu$ decay, (b) the predicted $f_{D_s^+}$ with theories based on QCD.

constants, which is $f_{D_s^+} = (247.5 \pm 2.2)$ MeV.

5.3 Comparison of measured and expected ratio of $f_{D_s^+}/f_{D^+}$

By weighting the decay constant f_{D^+} measured at the CLEO-c and BES-III experiments, we obtain $f_{D^+} = 205.3 \pm 5.1$ MeV. While the weighted average of the values of the measured D_s^+ decay constant is $f_{D_s^+} = 256.8 \pm 4.1$ MeV. With those two measured decay constants, we obtain the ratio $f_{D_s^+}/f_{D^+} = 1.251 \pm 0.037$. Figure 9 shows comparison of the predicted ratio $f_{D_s^+}/f_{D^+}$ with different theories based on QCD. The weighted average of these ratios is $f_{D_s^+}/f_{D^+} = 1.156 \pm 0.007$. At present, the measured ratio of $f_{D_s^+}/f_{D^+}$ is 2.5σ larger than the one predicted with theoretical calculations. This 2.5σ deviation of the measured ratio of $f_{D_s^+}/f_{D^+}$ from the predicted ratio with theories based on QCD may indicate that some effects of non-standard model enhance the decay rate of $D_s^+ \to l^+\nu$.

6 Determination of CKM matrix elements $|V_{\rm cd}|$ and $|V_{\rm cs}|$

The CKM elements $|V_{\rm cd}|$ and $|V_{\rm cs}|$ can be extracted from the leptonic decay branching fractions of the D^+ and D_s^+ mesons. In this section, we discuss the determinations of the $|V_{\rm cd}|$ and $|V_{\rm cs}|$ with the measured branching fractions for these leptonic decays and test of the unitary of the CKM matrix.

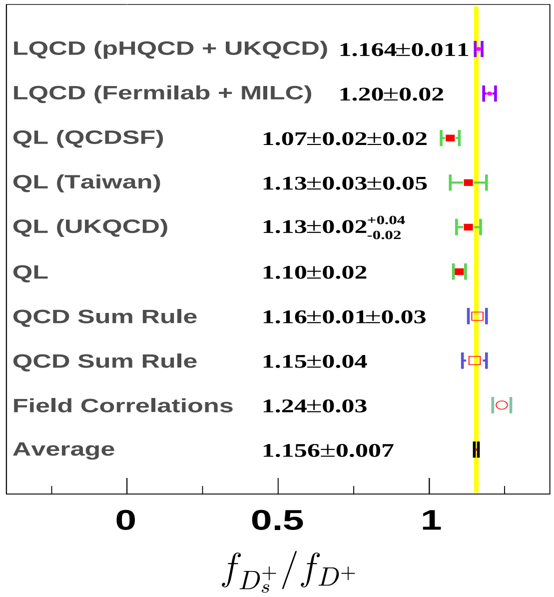

Figure 9: Comparison of the ratio of $f_{D_s^+}/f_{D^+}$ predicted with theories based on QCD.

6.1 Measurements of $|V_{\rm cd}|$ and $|V_{\rm cs}|$ with $D^+_{(s)} \to l^+\nu$

The BES-III determined the CKM matrix element $|V_{\rm cd}|$ with the measured branching fraction for $D^+ \to \mu^+\nu$ decay. Inserting the branching fraction measured at the BES-III, the mass of the muon, the mass of the D^+ meson, the decay constant $f_{D^+} = 207\pm 4$ MeV from LQCD [7], G_F and the lifetime of the D^+ meson [2] into Eq.(1) yields

$$|V_{\rm cd}| = 0.2218 \pm 0.0062 \pm 0.0047 \quad \text{(BESIII Preliminary)},$$

where the first error is statistical and the second systematic arising mainly from the uncertainties in the measured branching fraction (1.7%), f_{D^+} (1.93%), and the lifetime of the D^+ meson (0.7%) [2]. The total systematic error is 2.1%. Table 4 lists the comparison of the measured magnitude of $V_{\rm cd}$ from different experiments.

Table 4: Comparison of the measured $|V_{\rm cd}|$.

Experiment	$|V_{\rm cd}|$
PDG10 (Charm decays) [2]	$0.229 \pm 0.006 \pm 0.024$
PDG10 (ν and $\bar{\nu}$ interaction) [2]	0.230 ± 0.011
CLEO-c ($D \to \pi e^+\nu_e$) [42]	$0.234 \pm 0.007 \pm 0.002 \pm 0.025$
BES-III ($D^+ \to \mu^+\nu_\mu$)	$0.222 \pm 0.006 \pm 0.005$

With the $f_{D_s^+} = (247.5 \pm 2.2)$ MeV which is the weighted average of the predicted decay constants with theories based on QCD, we calculate the CKM matrix element $|V_{\rm cs}|$ with the branching fractions for $D_S^+ \to \mu^+\nu$ and $D_S^+ \to \tau^+\nu$ decays measured at different experiments.

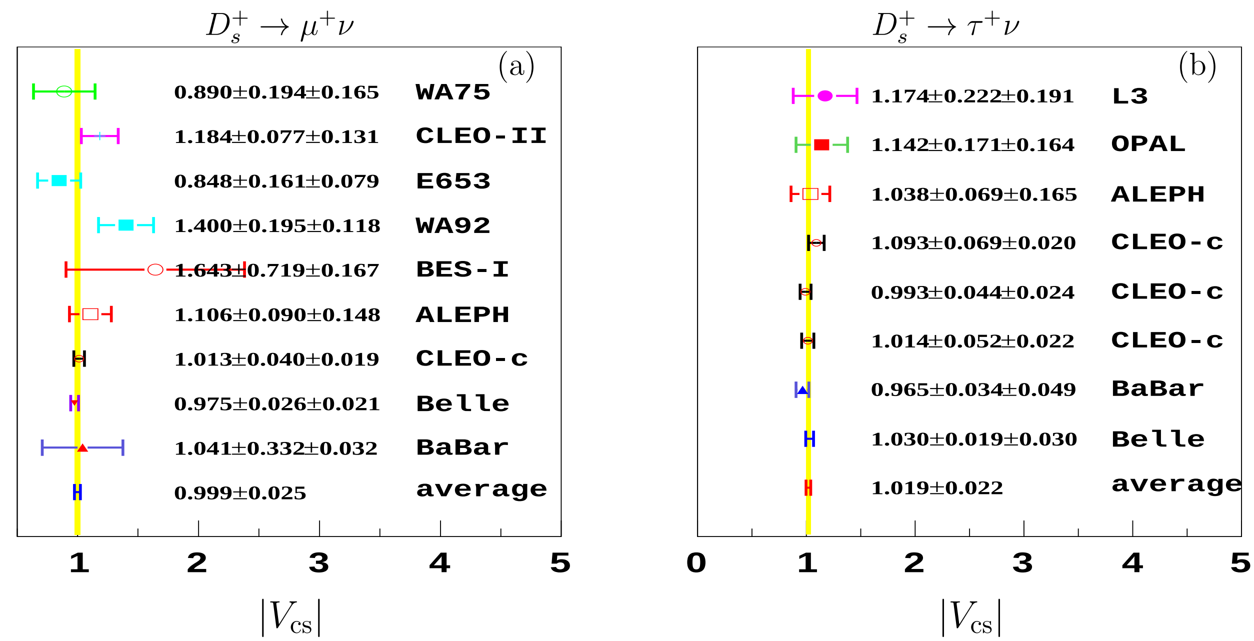

Figure 10: Comparison of the measured $|V_{\rm cs}|$ obtained from (a) $D_s^+ \to \mu^+\nu$ decay and (b) $D_s^+ \to \tau^+\nu$ decay.

Figure 10 (a) and (b) show comparison of the $|V_{\rm cs}|$ determined with decay branching fractions for $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ measured at different experiment, respectively. Figure 11 (a) shows the $|V_{\rm cs}|$ determined with decay branching fractions for $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ as well as the average of the $|V_{\rm cs}|$. The world average of $|V_{\rm cs}|$ is 1.010 ± 0.017.

With the $f_{D^+} = (212.7 \pm 3.2)$ MeV which is the weighted average of the predicted decay constants with theories based on QCD, we calculate the CKM matrix element $|V_{\rm cd}|$ with the branching fractions for $D^+ \to \mu^+\nu$ decay measured at the BES-III and the CLEO-c. Figure 11 (b) shows comparison of the $|V_{\rm cd}|$ determined with decay branching fractions for $D_s^+ \to \mu^+\nu$ measured at different experiment as well as the average of the $|V_{\rm cd}|$. The world average of $|V_{\rm cd}|$ is 0.2176 ± 0.0060.

Comparing the CKM matrix elements of $|V_{\rm cd}|$ and $|V_{\rm cs}|$ obtained by analyzing the D^+ and D_s^+ leptonic decays with those obtained by analyzing D meson semileptonic decays can provides some useful information about the New Physics beyond the Standard Model. If no nonstandard leptonic decay of the D_s^+ meson, the values of $|V_{\rm cd(s)}|$ determined from the D_s^+ leptonic decay branching fraction and determined from the $D_{(s)}$ meson semileptonic decay branching fractions should be the same. From the CKMfitter [2] one obtains $|V_{\rm cs}|_{\rm CKMfitter} = (0.97345^{+0.00015}_{-0.00016})$ and $|V_{\rm cd}|_{\rm CKMfitter} = (0.2252 \pm 0.0007)$. While from the leptonic decays of the D_s^+ and D^+ mesons we obtain $|V_{\rm cs}|_{D_s^+ \to l^+\nu} = (1.010 \pm 0.017)$ and $|V_{\rm cd}|_{D^+ \to l^+\nu} = (0.218 \pm 0.006)$. However, from D meson semileptonic decays, the CLEO-c measured $|V_{\rm cs}|_{\rm D\ semileptonic\ decays} = 0.985 \pm 0.009 \pm 0.006 \pm 0.103$ [42] and $|V_{\rm cd}|_{\rm D\ semileptonic\ decays} = 0.234 \pm 0.007 \pm 0.002 \pm 0.025$ [42]. From the values of these $|V_{\rm cd}|$ we found that the $|V_{\rm cd}|$ determined from D^+ leptonic decays is consistent within the error with these either determined from the D meson semileptonic decays or determined from the CKMfitter [2]. However,

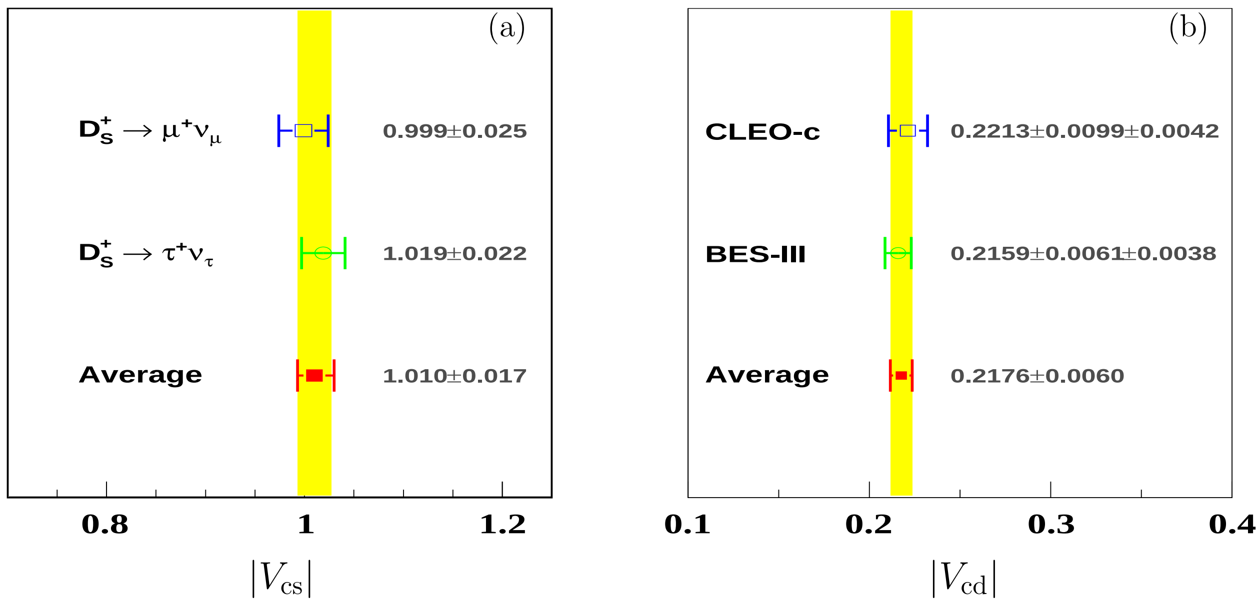

Figure 11: Comparison of the measured (a) $|V_{\rm cs}|$ obtained from $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ decays, and (b) $|V_{\rm cd}|$ obtained from $D^+ \to \mu^+\nu$ decay.

comparing the values of $|V_{\rm cs}|$ we found that the $|V_{\rm cs}|$ determined from the CLMfiter is consistent within error with the one determined from D semileptonic decays, but the value of the $|V_{\rm cs}|$ determined from D_s^+ leptonic decays is 2.2σ larger than the $|V_{\rm cs}|_{\rm CKMfitter}$. This 2.2σ deviation of $|V_{\rm cs}|_{D_s^+\to l^+\nu}$ from the $|V_{\rm cs}|_{\rm CKMfitter}$ may indicate that there are some New Physics effects which enhance the D_s^+ leptonic decays.

To make more precision comparison of these CKM matrix elements, we need to reduce the errors of the measured decay branching fractions of D_s^+ and D^+ leptonic and semileptonic decays as well as reduce the errors of the measured decay branching fractions of the D^0 semileptonic decays. These could well be done at the currently running BES-III experiment and at the BELLE and BaBar experiments. At present, two analysis working groups (IHEP and CMU groups) [43] in the BES-III collaboration have been working on extracting the $|V_{\rm cs}|$ and $|V_{\rm cd}|$ as well as other physical quantities from D meson semileptonic decays. Based on these two working group analysis of D^0 semileptonic decays with a portion of data taken at 3.773 GeV, the BES-III collaboration report preliminary results on measurements of $|V_{\rm cs}|f_+^K(0)$ and $|V_{\rm cd}|f_+^\pi(0)$ [43], where $f_+^K(0)$ and $f_+^\pi(0)$ are the form factors D semileptonic decays. More precision measurements of $|V_{\rm cs}|$ and $|V_{\rm cd}|$ would be important physics results for precise test of the SM and search for New Physics.

7 Conclusion and Outlook

Since the first attempt to search for the D^+ leptonic decay, although no signal event was found for this decay performed at the MARK-III experiment in 1988, many experiments have been making great efforts to search for and study the D^+ and D_s^+ leptonic decays. After

more than 25 year studies of D^+ and D_s^+ leptonic decays, more than 530 $D^+ \to \mu^+\nu$ and about 4×10^3 $D_s^+ \to l^+\nu$ decay events have been found. One begins to precisely study the hadronic vertex of the D^+ and D_s^+ meson decays and precisely test the LQCD calculations of the decay constants f_{D^+} and $f_{D_s^+}$. At the Charm2012 conference, the BES-III collaboration report the most precise results for measurements of the decay branching fraction, decay constant and $|V_{\rm cd}|$ in the world, which are $B(D^+ \to \mu^+\nu) = (3.74 \pm 0.21 \pm 0.06) \times 10^{-4}$, $f_{D^+} = (203.9 \pm 5.7 \pm 2.0)$ MeV, and $|V_{\rm cd}| = (0.222 \pm 0.006 \pm 0.005)$. The most precise measurements of $B(D_s^+ \to l^+\nu)$ and $f_{D_s^+}$ are from the BELLE experiment. The BELLE results are $B(D_s^+ \to \mu^+\nu) = (0.528 \pm 0.028 \pm 0.019)\%$, $B(D_s^+ \to \tau^+\nu) = (5.70 \pm 0.21^{+0.31}_{-0.30})\%$, and $f_{D_s^+} = (255.0 \pm 4.2 \pm 5.0)$ MeV.

The world average of decay constants are $f_{D^+} = (205.3 \pm 5.1)$ MeV and $f_{D_s^+} = (256.8 \pm 4.1)$ MeV. The two decay constants yield the world average of the ratio of $f_{D_s^+}/f_{D^+} = 1.251 \pm 0.037$, which is about 2.5σ larger than $f_{D_s^+}/f_{D^+} = 1.156 \pm 0.007$ predicted with theories based on QCD. By comparing the $|V_{\rm cs}|$ and $|V_{\rm cd}|$ determined from the D_s^+ and D^+ meson leptonic decays, determined from the CKMfitter [2], and determined from the D meson semileptonic decays, we also found that the $|V_{\rm cs}|$ determined from D_s^+ leptonic decays is 2.2σ larger than the $|V_{\rm cs}|_{\rm CKMfitter}$. This 2.2σ deviation of $|V_{\rm cs}|_{D_s^+ \to l^+\nu}$ from the $|V_{\rm cs}|_{\rm CKMfitter}$ may indicate that there are some New Physics effects which enhance the D_s^+ leptonic decays.

The measured D^+ and D_s^+ meson decay constants f_{D^+} and $f_{D_s^+}$ can be used to test LQCD calculations of the decay constants. At present the uncertainties of the measured decay constants are almost the same as the uncertainties of the LQCD calculations of the decay constants. To more precisely test the LQCD calculations of the decay constants, we still need more data to be collected at 3.773 GeV, at energy near $D_s^+ D_s^-$ meson pair production energy threshold, and at the higher energy of about 10.6 GeV. These data taking will be performed at the BES-III and other B factory experiments in the future. The verified LQCD calculation help extract $|V_{\rm td}|$ and $|V_{\rm ts}|$ from $B\bar{B}$ mixing experiments. These help more precisely test the SM and search for New Physics beyond the SM.

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